Why Statisticians Have the Worst Luck

Or, Why Not to Travel with a Statistician

by Paul Mullenix, Ph.D.

Most people do not like statisticians, or at least would not like to be a statistician. How would you like to be able to prove that you were right in this assessment?

I am a statistician and recently I was attending a conference with my boss. During this trip, I invariably chose the longest waiting lines at tolls, the slowest lanes of traffic, the longest check-out lines and the longest queues for food service. My boss attributed this to my being a statistician. Perhaps she was right—you can be the judge.

The argument in this article that no one has worse luck than a statistician derives from a famous probabilitist, William Feller. According to Feller (1),

> "As everybody knows, he who joins a waiting line is sure to wait for an abnormally long time, and similar bad luck follows us on all occasions."

To understand why this might be true, let x_0 be the waiting time (or financial loss, or some other calamity) experienced by a statistician during some chance event. Let x_1, x_2, … be the results for others experiencing the same chance event. We assume that all the results are independent and identically distributed. Our question can now be phrased more technically as,

> How long will it be, on the average, before someone experiences worse luck than the statistician?

To answer this, let N be the event that someone else does have worse luck than the statistician. That is, for our waiting line example, N=n is the event that $x_n > x_0$ and n is the smallest subscript for which this is true (i.e. the nth person is the first such person who has waited longer than the statistician).

Thus, the event that we have to wait more than n-1 people until someone waits longer than the statistician means that the maximum waiting time among persons 0, 1, 2, …, n-1 is the statistician. If N is to be greater than n-1, then the statistician must have the worst luck among the first n-1 people. Writing this mathematically we have

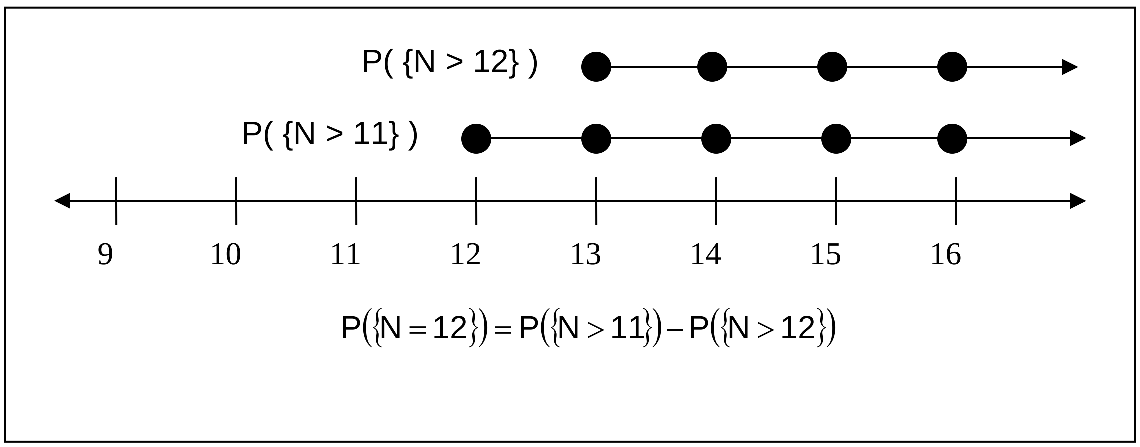
$$\{N > n-1\} = \{\max\{x_0, x_1, x_2, \ldots, x_{n-1}\} = x_0\}. \qquad (1)$$

Since the x's, or waiting times, are independent and identically distributed, any of the persons 0, 1, 2, …, n-1 are equally likely to have the maximum time. Thus the chance that the statistician has the worst luck is 1/n, where n is the number of people in the line.

Stating this in probability symbols means that

$$P(\{N > n-1\}) = \frac{1}{n}. \tag{2}$$

Figure 1. Illustration of how to Compute P({N = 12}) , the Probability that the 12th Person has the Worst Luck.



From Figure 1 we see that the probability that {N=12} is the probability that N is greater than 11 less the probability that N is greater than 12. It is evident that we may apply formula (2) to the relationship in Figure 1 to compute the general expression,

$$\begin{aligned} P(\{N = n\}) &= P(\{N > n-1\} - P(\{N > n\} \\ &= \frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)} \end{aligned} \tag{3}$$

The answer to our question, the expected time until someone has worse luck than the statistician, can now be computed by multiplying each value of N by the probability that N takes that value and summing these as follows.

$$\text{Average of } N = \sum_{n=1}^{\infty} n\, P(\{N = n\}) = \sum_{n=1}^{\infty} n \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n+1} = \infty \tag{4}$$

This last sum on the right of expression (4) is the famous harmonic series and diverges to infinity! Hence on the average, we would have to wait for an infinite number of people until someone had worse luck than the statistician! Or in other words, no one has worse luck than the statistician.

Notice that this assumes there are an infinite number of people in line. If there were only 1000 people visiting the waiting line in a day, then formula (4) would become,

$$\sum_{n=1}^{1000} \frac{1}{n+1} \approx 6.48 \,. \tag{5}$$

According to expression (5), one would only have to wait a little over 6 people, on the average, to find someone with worse luck. So there is still hope for the unfortunate statistician.

Moral: You are right to assume that no one has worse luck than the statistician. But it is only the statistician who would bother to prove that he has the worst luck.

(1) Feller, William, *An Introduction to Probability Theory and Its Applications, Vol. II*, John Wiley & Sons, Inc., New York 1971, p. 15.